Financial Data on Markowitz Model and Index Model

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Abstract: Calculating the portfolios in both Markowitz and single-index models are significant in investing market. Nevertheless, such literature has not been applied successfully in the real life. Two models are established by solving and graphing the minimum variances frontier, efficient frontier, maximum return frontier, max Sharpe ratio, minimum variances, as well as the CAL. The empirical results indicating that the Markowitz model is much more accurate and precise than the index model since the Markowitz model considers more factors than the index model. However, the index model also has its advantage: the index model can help people get the estimated data that is similar to the Markowitz model in a short time, and the index model is accurate enough for investors to use. Two models are established to help investors get the optimal portfolios and several options according to investors' level of risk-aversion in different conditions. This paper provides a better understanding of two models in both concept and calculation parts.

1. Introduction

When investing in securities and other risky assets, two core issues need to be addressed first: expected return and risk. So how to measure the risk and return of portfolio investment and how to balance these two indicators to allocate assets is an urgent problem for market investors to solve. It was against this background that, in the 1950s and early 1960s, the Markowitz theory was born.

Markowitz theory attracts many attentions in academic research. Mangram [1] presents a simplified perspective of Markowitz' contributions to Modern Portfolio Theory, foregoing in-depth presentation of the complex mathematical/statistical models typically associated with discussions of this theory, and suggesting efficient computer-based 'short-cuts' to these performing these intricate calculations. Love [2] attempts to develop a model, along the lines of the Markowitz model of portfolio analysis, which would allow one to investigate the impact of diversification. The analysis suggests that diversification does not invariably reduce or eliminate export fluctuations and that diversification may involve a cost in terms of lost export earnings. Boyle [3] thinks this model shows that for any given level of expected returns, the optimal portfolio depends on two quantities: relative ambiguity across assets and the standard deviation of the expected return estimate for each asset. If both quantities are low, then the optimal portfolio consists of a mix of familiar and unfamiliar assets; moreover, an increase in correlation between

Assets causes an investor to increase concentration in familiar assets (flight to familiarity).

For index mode, Hadley [4] states this model also provides new insights into waveguiding phenomena in vertical-cavity lasers. In particular, it is shown that the effective index responsible for waveguiding is dependent only on lateral changes in the Fabry–Perot resonance frequency. This concept leads naturally to new design methods for these lasers that are expected to result in more efficient devices with superior modal characteristics. Hao [5] thinks that a multivariate, multi-index drought-modeling approach is proposed using the concept of copulas. The proposed model, named

Multivariate Standardized Drought Index (MSDI), probabilistically combines the Standardized Precipitation Index (SPI) and the Standardized Soil Moisture Index (SSI) for drought characterization. Stone [6] says that this constraint is used as information to increase the accuracy of the confidence regions (smaller regions at the same nominal level). As a by-product, our approach of using bias correction may also shed light on nonparametric estimation in model checking for other semiparametric regression models.

This research established two models to make the portfolio for 7 different stock (SPX, NVDA, CSCO, XOM, CVX, KO, MCD). Minimum variances are considered, which is also the risks, and the maximum Sharpe ratio for 7 stocks understand three different conditions to find the optimal portfolio. By using the solver table, the efficient frontier can be determined, variance frontier, and efficient frontier which are necessary for us to create the different combinations of portfolios. The values from the solver table to graph the two models can be used. As there are three conditions, we need to get three different graphs for each model. For the first condition, nothing needs to be changed because it is free of any constraint. For the second condition, however, a constraint is needed to be set in the solution that all the weight need to be greater or equal to 0 since we are not allowed to have any kind of short position which means we cannot have any stock that weight less than 0. The first stock need to be set to 0 because we want to see if the inclusion of the broad index has a positive or negative impact on the portfolio, and the data is needed to be analysed and contrasted from condition 1 (with the broad stock) and condition 2 (no broad stock) to get the result. After inserting all the information from the solver table, maximum variances points and maximum Shape ratio points also need to be graphed. Meanwhile, we also need to find the free risky asset and connect the free risky asset to the Maximum Sharpe ratio to help us get the Capital Allocation Line.

The remainder of the paper is organized as follows: Section 2 describes the data and graphs; section 3 performs the specific procedures of establishing two models; section 4 the conclusive analysis of the outcomes of two models. The last section is the conclusions for the research.

2. Data

As shown in Figure 1, the price of SPX is collected from January 1, 2000 to January 1, 2020. As can be seen from Figure 1, the trend of SPX is rising, but in 2008, the price dropped from about 1,500 yuan per share to about 500 yuan per share.

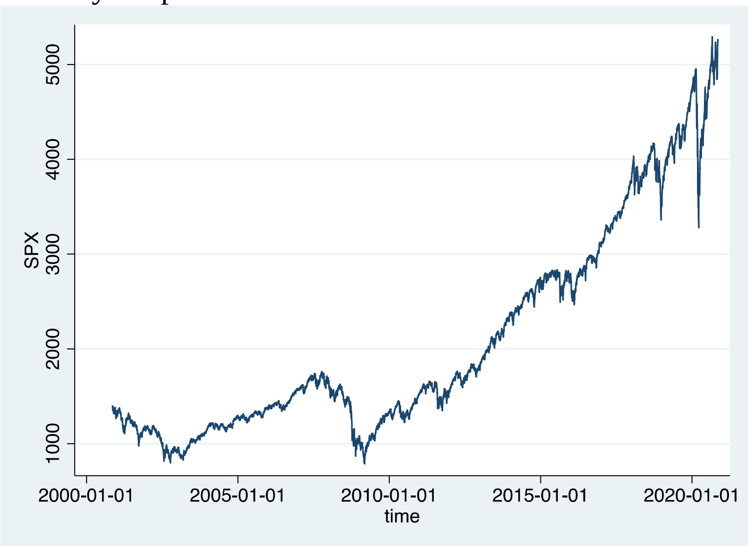

Figure 1. The price trend of SPX

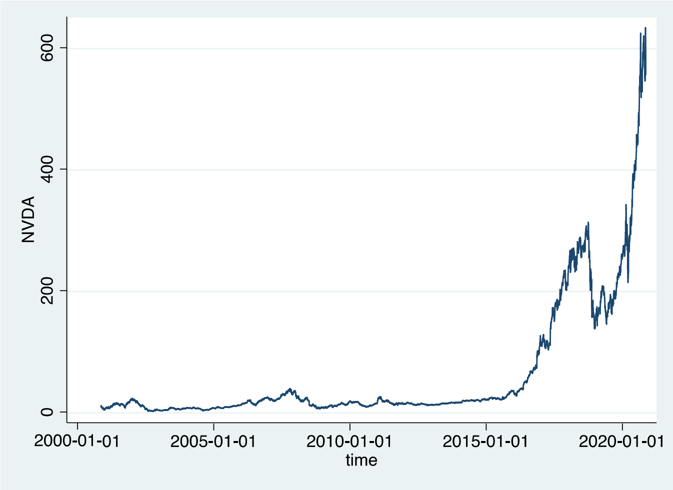

Figure 2. The price trend of NVDA

Figure 2 represents the NVDA prices from January 1, 2000 to January 1, 2020. Before 2016, its price almost maintained at about 20 yuan per share. However, after 2016, the stock price showed an upward trend, and in 2018, it dropped from 300 yuan per share to about 180 yuan per share.

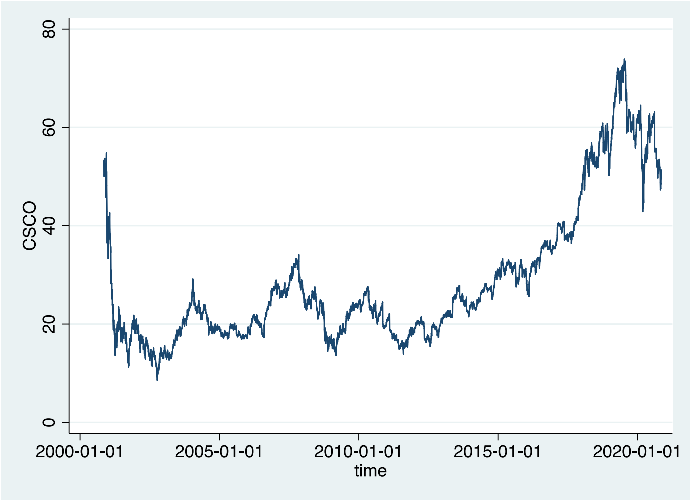

Figure 3. The price trend of CSCO

Figure 3 shows the CSCO prices from January 1, 2000 to January 1, 2020. In 2000, its price was around $50 per share, and then it fell to around $20 per share. From 2002 to 2017, its price fluctuated around 20 to 40 yuan per share, but from 2017 to 2018, its price showed an upward trend

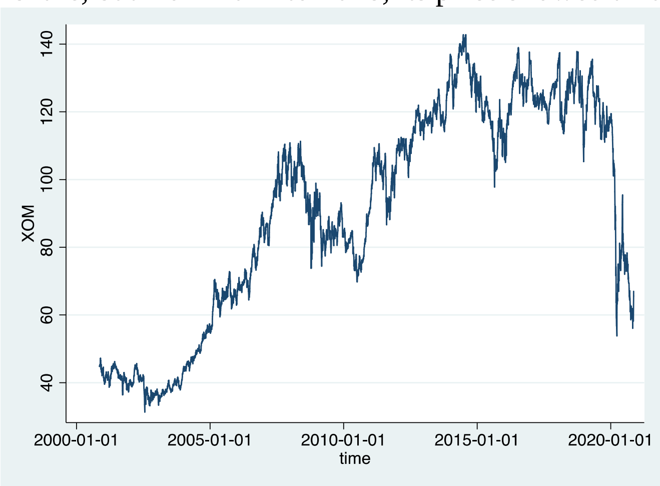

Figure 4. The price trend of XOM

As shown in Figure 4, the XOM price is collected From January 1, 2000 to January 1, 2020. From 2000 to 2008, this stock showed an upward trend. From 2008 to 2010, its price dropped from about 110 yuan per share to about 70 yuan per share. From 2010 to 2015, it showed an upward trend, but on January 1st, 2015, it dropped from about 140 yuan per share to 110 yuan per share. In 2016, the price rose to about 140 yuan per share, which was relatively stable before 2020, but in 2020, the price dropped from about 120 yuan per share to about 60 dollars per share.

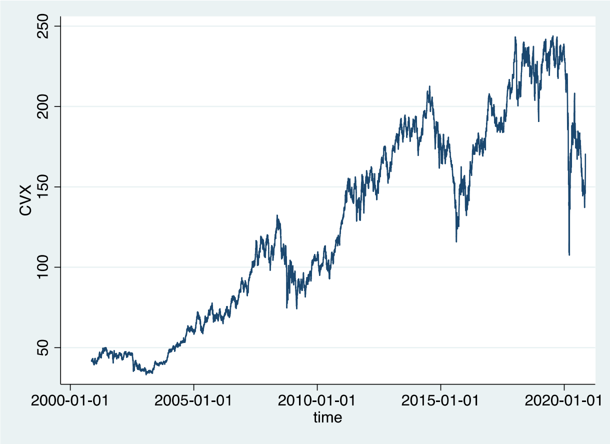

Figure 5. The price trend of CVX

As Figure 5 shown, the price of CVX is collected from January 1, 2000 to January 1, 2020. The overall trend was upward, but prices plunged around 2016 and 2019.

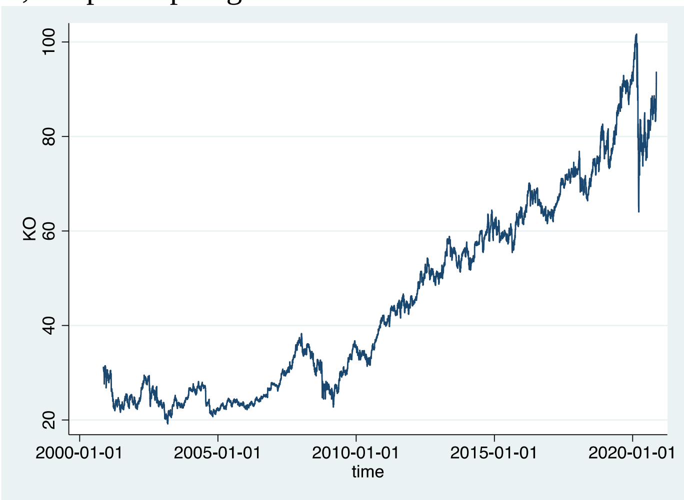

Figure 6. The price trend of KO

As shown in Figure 6, the prices of KO is collected from January 1, 2000, to January 1, 2020. The price of KO has been on the rise, but in 19 years or so, the price dropped from 100 yuan per share to 70 yuan per share.

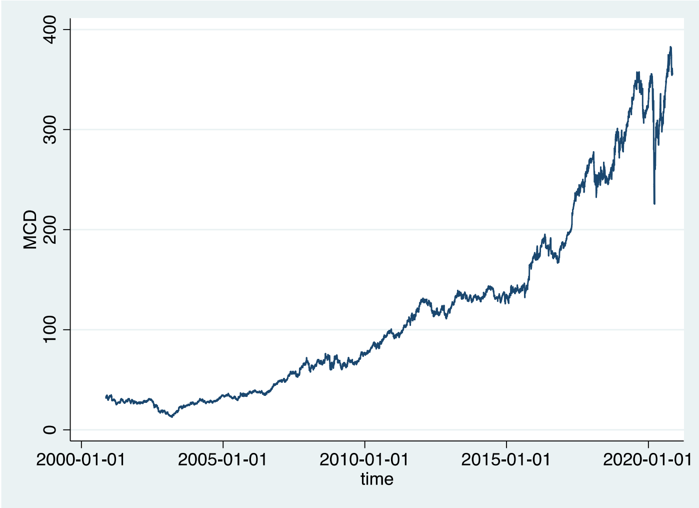

Figure7 the price trend of MCD

Figure 7 represents the MCD prices from January 1, 2000 to January 1, 2020. Its price followed the trend of shopping malls, but in 2020, its price dropped from about 300 yuan per share to about 220 yuan per share, and then rose back to about 300 yuan per share.

Table 1. A basic introduction to stocks

	SPX	NVDA	CSCO	XOM	CVX	KO	MCD
avg return	6.651%	35.535%	4.314%	2.884%	8.251%	5.773%	12.833%
stdev	15.075%	58.919%	33.010%	19.869%	21.582%	16.532%	19.036%
annualized beta	100.000%	199.316%	140.037%	76.437%	86.711%	55.035%	65.061%
annualized alpha	0.000%	22.277%	-5.000%	-2.200%	2.484%	2.112%	8.505%
annualized residuals stdev	0.000%	50.682%	25.377%	16.187%	17.173%	14.299%	16.315%

As Table 1 , the average return of SPX is 6.651%,the average return of NVDA is 35.535%,the average return of CSCO is 4the average return of 4.314%,the average return of XOM is 2.884%, the average return of CVX is 8.251%,the average return of KO is 5.773%,the average return of MCD is 12.833%. The stdev of SPX is 15.075%,the stdev of NVDA is 58.919%,the stdev of CSCO is 33.010%,the stdev of XOM is 18.869%, the stdev of CVX is 21.582%,the stdev of KO is 15.532%, the stdev of MCD is 19.036%. other information of stocks could be found in this table.

3. Method

For the group's data constraints. The first one is a free constraint, which means that there are no constraints. The second constraint is that you cannot sell those stocks to make the data negative or you did not buy any stocks. The last one is the first stock's index is equal to zero.

Moreover, for Markowitz Model theory, simply means that is a mathematical framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk. It is a formalization and extension of diversification in investing, the idea that owning different kinds of financial assets is less risky than owning only one type. Its key insight is that an asset's risk and return should not be assessed by itself, but by how it contributes to a portfolio's overall risk and return. It uses the variance of asset prices as a proxy for risk.

Index Model: The single-index model (SIM) is a simple asset pricing model to measure both the risk and the return of a stock. The model has been developed by William Sharpe in 1963 and is commonly used in the finance industry. Mathematically the SIM is expressed as:

The Single Index Model. Relates returns on each security to the returns on a common index, such as the S&P 500 Stock Index.

Expressed by the following equation

$R_i = \alpha_i + \beta_i R_M + e_i$ (1)

Dives return into two components
-a unique part,α_1
-a market-related part,$\beta_1.R_M$
where:
In the application of those two models, and the analysis shows that compare Markowitz Model to Index Model, Markowitz Model is more convenient to provide researchers valuable data and results.

4. Result Analysis

This section analyzes the information and data from two models according to the differences and similarities between the two models and different constraints.

4.1Markowitz model

In the Markowitz model, the different portfolios are calculated to understand three different conditions. The investors are supposed to be risk-averse, and they want to take as much profit as possible which means the maximum Sharpe ratio is needed to calculate in this model. Thus, we first pull out the data of 7 stocks and calculate the maximum Sharpe ratio and the minimum risk (variances) by using the solution. Meanwhile, the efficient frontier is needed to be found, minimum variance frontier, minimum return frontier, and the CAL line to help the investor make the decision. After get the model, the investor can always make their portfolio according to their level of risk-averse.

Table 2. Data results for Markowitz model.

		Expected return	Expected StDev	Sharpe Ratio
c1	Min Var	5.230%	12.964%	0.403
	Max Sharpe	31.112%	31.619%	0.9839577
c2 w>0	Min Var	6.723%	13.315%	0.505
	Max Sharpe	17.79%	20.94%	0.84925951
c3 w1=0	Min Var	6.154%	14.121%	0.436
	Max Sharpe	31.370%	31.883%	0.9839215

Table 3. Weights of stocks for Markowitz model.

		SPX	NVDA	CSCO	XOM	CVX	KO	MCD
c1	min variance	42.94%	-5.73%	-9.21%	13.32%	6.67%	32.57 %	19.44 %
	max Sharpe	- 11.63%	22.40 %	- 20.06%	- 21.69%	21.75 %	26.68 %	82.53 %
c2 w>0 or=0	min variance	20.56%	0.00%	0.00%	14.70%	7.36%	35.94 %	21.45 %
	max Sharpe	0.00%	16.89 %	0.00%	0.00%	6.75%	12.31 %	64.05 %
c3 w1=0	min variance	0.00%	-4.05%	-3.96%	22.41%	15.30 %	42.58 %	27.72 %
	max Sharpe	0.00%	21.44 %	- 21.13%	- 23.24%	19.34 %	24.34 %	79.26 %

4.1.1 Index model

Although the index model looks different from the Markowitz model since it only considers about one Marco factor, during the process of calculating the data, it is the same as the Markowitz model. Repeating the procedures of the Markowitz model can help us get the index model.

Table 4. Data results for index model.

		Expected Return	Expected StDev	Sharpe Ratio
c1	min variance	5.733%	12.169%	0.471
	max sharpe	19.623%	22.514%	0.872
c2 w>0 or=0	min variance	7.226%	12.783%	0.565
	max sharpe	15.487%	19.045%	0.813
c3 w1=0	min variance	6.315%	12.646%	0.499
	max sharpe	19.207%	22.054%	0.871

Table 5 Weights of stocks for index model.

		SPX	NVDA	CSCO	XOM	CVX	KO	MCD
c1	min variance	0.429355	-0.05726	-0.09207	0.133183	0.066731	0.325672	0.19439
	max Sharpe	-0.11627	0.224018	-0.20056	-0.21688	0.217532	0.266808	0.82535
c2 w>0 or=0	min variance	0.205562	0	0	0.146957	0.073632	0.359355	0.214495
	max Sharpe	0	0.168867	0	0	0.06752	0.123149	0.640464
c3 w1=0	min variance	0	-0.04047	-0.03956	0.224115	0.152984	0.425751	0.277183
	max Sharpe	0	0.214372	-0.21134	-0.23236	0.193366	0.243357	0.792614

4.2 Comparison between different models Index model

According to the data in all tables, in condition 2, the Markowitz model has more stocks that weigh equal to 0 than in the single-index model. Also, the maximum Sharpe ratio and expected return in the Markowitz model are generally smaller than that in the index model. This is happening because the Markowitz model have to do more calculation than the index model, and they will get less volatility. Also, they have more accurate data than the index model. The difference between expected standard deviations in the two models can also be explained in this way: since the index model only consider one macro factor, the index model would have fewer things to consider which means fewer fluctuations, and they can get an estimated data.

5. Conclusion

In this study, the results show that the Markowitz model will choose at most a third of the number of stocks available in the index to construct a portfolio. For both the financial and industrial sectors, not more than half of the stocks will be included in the optimal portfolio of two different models. The different ways of selecting stocks resulted in different returns generated under each model.

This research compares two models into two different aspects: differences in constraints and the model itself. According to the process of making two models, as well as the data from two models, the risk of condition 1 is generally smallest in two models compare with the other two conditions, and its Sharpe ratio is overall the highest among different. The index will have a positive effect on the portfolio in condition 3. For the graph of condition 2, it will intersect at a point. However, for the graph of condition 1 and 3, the lines of them are opening to two sides because the shorting position is allowed.

The index model is more adept to the market since the index model only considers only one factor, people do not need to do many calculations.

Regarding the limitations, the first thing that needs to be considered is that there is not enough data and practice to reduce the error of the conclusion. In our experience and analyses, there are only three limits and few variables, those conditions can inevitably lead to some inaccuracy of this research.

For future study plan, since only limited concepts are understood, learning how to apply them, in reality, is very vital. Therefore, the next step will focus on how to utilize the Markowitz Model and Index Model in some real business cases and financial problems.

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